A Fourier-Mellin based Equalization Technique for Wideband Linear Time Varying Channels

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Abstract—**We wish to suggest a novel equalization technique for wideband linear time varying channels. Motivated from the mammalian auditory system, the central idea behind the equalization strategy is a combination of Mellin and Fourier transforms which is able to compensate the concurrent Doppler and delay spread introduced by the channels. The proposed method does not require large memory usually associated with the buffering process in conventional approach while being computationally attractive simultaneously. Mathematical development of the proposed system is presented along with simulations demonstrating the effectiveness of the same.**

Keywords—*Wideband Channels; Mellin Transform; Channel Equalization; Underwater Acoustic Communication; Bio-Inspired*

I. INTRODUCTION

Establishing a reliable high speed communication link over a wideband linear time varying (WB-LTV) channel is quite challenging. Such channels are encountered whenever there is fast relative motion between transmitter, receiver and/or scatters present in the environment, like in underwater acoustic communication [1] and vehicular communication [2]. This relative motion leads to simultaneous spreading in time and frequency, in addition to the usual difficulties faced in typical narrowband communication like multipath and inter-symbol interference (ISI), making the system design much more convoluted [3]. Thus the major additional challenge lies in tackling the large frequency dependent Doppler shifts, for which typical approach taken by usual commercial systems is to employ a non-coherent modulation, but substantially limiting the data rate [4]. Newer approaches reported in the literature aiming to attain still higher data rate over the WB-LTV channels, can be divided into two broad categories: one based on extension of narrowband concepts and other based on exploitation of time-scale diversity. In the former technique, the general procedure is to employ a multicarrier modulation (like OFDM, OWDM) followed by equalization [1, 5, 6] and in the later one, the use of RAKE receiver which is generalized to a time-scale version is proposed [6, 7, 8].

In the present work, a communication system based upon orthogonal wavelet division multiplexing (OWDM) is considered, similar to [6]. (For OWDM over AWGN channels the readers are referred to [9, 10].) This scheme unifies the above mentioned approaches as the multi-scale orthogonal wavelets behave as multicarrier modulation as well as take advantage of time scale diversity and also they are spectrally efficient. However, instead of employing a computationally intensive RAKE type of receiver or conventional equalizers

like LMMSE, an alternative approach inspired from mammalian hearing is adopted by us. It is motivated from the observation that mammals do not face a problem in recognizing a Doppler scaled or time lagged sound (or word in case of humans). Upon study, it has been inferred that a combination of Mellin Transform, an integral transform having the desirable property of scale invariance (up to a phase shift), and Fourier Transform must be occurring in the auditory system [11]. This idea is incorporated in the proposed equalization technique. Similar borrowing has been carried out in the field of image processing to extract rotation, scale and translation invariant features [12] and in radar signal processing to identify simultaneously speed and location of the target [13]. In such applications typically only a single scaling and time shifting is relevant and the combination of Mellin and Fourier transform were employed successfully to combat this. But in the case of WB-LTV channels the scenario is more intricate due to the presence of multiple Doppler scales and time lags, yet the present work utilizes a combination of Mellin and Fourier transform to overcome this twin difficulties posed by WB-LTV channels.

The remainder of this paper is organized as follows: In section II background of Mellin Transform is given. Section III deals with the wideband channel model and appropriate simplification and discretization. System design and implementation details are discussed in Section IV. The simulation results are presented in section V and finally, concluded in section VI.

II. MELLIN TRANSFORM

Mellin transform is a linear integral transform that maybe regarded as a Fourier transformation on the multiplicative group of positive real numbers (i.e., group of dilations) and its development parallels the group-theoretical presentation of the usual Fourier transform [14]. Now if $\phi(f)$ is an analytical signal (i.e. $\forall f < 0 : \phi(f) = 0$) in the space of finite-energy signals $L^2(\mathbb{R})$, then its Mellin transform is defined by:

$$\mathfrak{M}[\phi](\beta) = \int_0^\infty \phi(f) f^{r+j2\pi\beta} df \qquad (1)$$

and the inverse Mellin transform is given by:

$$\phi(f) = \int_{-\infty}^\infty \mathfrak{M}[\phi](\beta) f^{-r-1-j2\pi\beta} d\beta \qquad (2)$$

For further information regarding Mellin transform the reader is referred to a more in-depth treatment like [14]. Here only the relevant property of Mellin transform, from [14],

sought to be exploited in equalization of WB-LTV channel are stated below:-

Property I – Scaling: The scaled function defined as:
$$\mathcal{D}_a[\phi](f) = a^{r+1}\phi(af) \tag{3}$$
is transformed according to:
$$\mathfrak{M}[\mathcal{D}_a[\phi]](\beta) = a^{-j2\pi\beta}\mathfrak{M}[\phi](\beta) \tag{4}$$
This scaling property is analogous to the translation property of Fourier transform.

Property II – Multiplicative Convolution: The Mellin transform of the multiplicative convolution of two functions $\phi(f)$ and $\psi(f)$ defined as:
$$(\phi ** \psi)(f) = \int_0^\infty \phi(\eta)\psi\left(\frac{f}{\eta}\right)\frac{d\eta}{\eta} \tag{5}$$
is equal to the product of their Mellin transforms:
$$\mathfrak{M}[\phi ** \psi](\beta) = \mathfrak{M}[\phi](\beta)\mathfrak{M}[\psi](\beta) \tag{6}$$
This multiplicative convolution property is identical to the convolution property of Fourier Transform.

A geometric sampling theorem involving Mellin Transform parallel to the arithmetic Nyquist-Shannon sampling theorem involving Fourier Transform has been established [15]. Suppose the Mellin transform of the signal $\phi(f)$ has a bounded support $[-1/2\ln q, 1/2\ln q]$, then the signal can be uniquely reconstructed from the samples taken at the geometric sampling rate of q as:

$$\phi(f) = \frac{1}{f^{r+1}}\sum_k q^{k(r+1)}\phi(q^k)\frac{\sin\pi\left(\frac{\ln\eta}{\ln q} - k\right)}{\pi\left(\frac{\ln\eta}{\ln q} - k\right)} \tag{7}$$

Further a discretization of Mellin Transform has been developed [16], alike the discrete Fourier Transform. Suppose the signal $\phi(f)$ and its Mellin transform, both have a bounded support of $[1, q^B]$ and $[-1/2\ln q, 1/2\ln q]$ respectively, then forward discrete transform of is given as:

$$\mathfrak{M}[\phi]\left(\frac{p}{B\ln q}\right) = \sum_{k=0}^{B-1} q^{k(r+1)}\phi(q^k)e^{j2\pi pk/B} \tag{8}$$

and the reverse discrete transform as:

$$\phi(q^k) = \frac{q^{-k(r+1)}}{B}\sum_{p=0}^{B-1}\mathfrak{M}[\phi]\left(\frac{p}{B\ln q}\right)e^{-j2\pi pk/B} \tag{9}$$

Thus, it is evident that discrete Mellin transform can be implemented with a Fast Fourier Transform (FFT) algorithm.

III. WIDEBAND CHANNEL MODEL

In general a WB-LTV channel can be expressed as [3]:

$$y(t) = \int_0^\infty \int_{-\infty}^\infty h_{WB}(\eta,\tau)\eta^{r+1}x(\eta(t-\tau))d\tau d\eta + v(t) \tag{10}$$

Here the received signal $y(t)$ can be considered as a continuous dense superposition of differently delayed (by τ) and scaled (by η) versions of the transmitted signal $x(t)$ weighted by the wideband channel spreading function $h_{WB}(\eta,\tau)$ along with the additive white Gaussian noise $v(t)$. Note that in (10) η^{r+1} is simply a normalization factor. Now for a practical channel, τ and η can be considered to be limited to $\tau \in [0, \tau_{max}]$ and $\eta \in [\eta'_{min}, \eta'_{max}]$. Without loss of generality, the scale can be restricted to $[1, \eta_{max}]$ by appropriately delaying and scaling
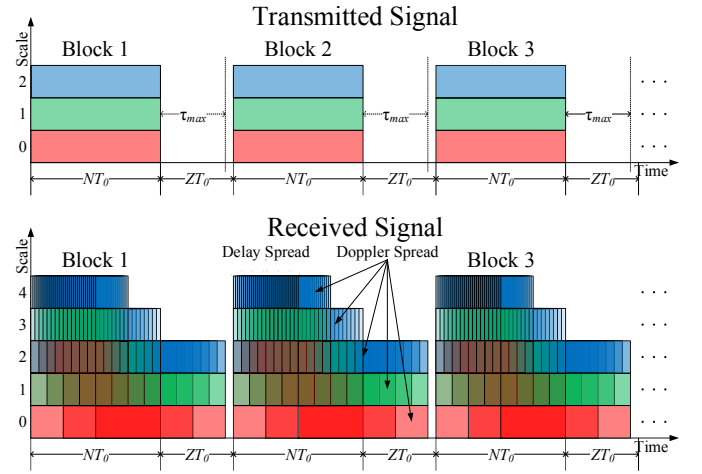

Figure 1. Transmission block structure and visualization of degradation caused by MSML channel.

the received signal by constants. The parameters τ_{max} and η_{max} respectively represent the *delay spread* and *Doppler scale spread*. A special case consists of separable spreading function, i.e. independent scale (Doppler) and delay power profile: $h_{WB}(\eta,\tau) = h_s(\eta)h_d(\tau)$. This reasonable assumption includes popular wide sense stationary uncorrelated scattering (WSSUS) wideband channel models like the frequently used one with exponential delay power profile and Jakes Doppler power profile [17] or the worst case statistic representing brick spreading function [18]. With this assumption the received signal can be expressed as:

$$y(t) = \int_1^{\eta_{max}}\int_0^{\tau_{max}} h_s(\eta)h_d(\tau)\eta^{r+1}x(\eta(t-\tau))d\tau d\eta + v(t) \tag{11}$$

An equivalent discrete model of the WB-LTV channel can be constructed as in [19]:

$$y(t) = \sum_{k=0}^{L_s}\sum_{l=0}^{L_d(k)} \tilde{h}_s[k]\tilde{h}_d[k,l]q^{k(r+1)}x(q^k t - lT) + v(t) \tag{12}$$

when the transmitted signal $x(t)$ has bandwidth of $1/T$ and Mellin support of $1/\ln q$. This be can identified with multi-scale and multi-lag (MSML) model of [20], which is a generalization of tapped delay line model. In such models, in addition to delay line taps there are scale taps for each delay. In (12), the number of scale channel taps is denoted by $L_s = \lceil \ln\eta_{max}/\ln q \rceil$, number of channel taps by $L_d(k) = \lceil q^k\tau_{max}/T \rceil$. Also $\tilde{h}_s[m]$ and $\tilde{h}_d[n]$ represents the so called smoothed spreading function (SSF) [7, 19], given by:

$$\tilde{h}_s[k] = \int_1^{\eta_{max}} h_s(\eta)\frac{\sin\pi\left(\frac{\ln\eta}{\ln q} - k\right)}{\pi\left(\frac{\ln\eta}{\ln q} - k\right)}d\eta$$

$$\tilde{h}_d[k,l] = \int_0^{\tau_{max}} h_d(\tau)\frac{\sin\pi\left(\frac{\tau q^k}{T} - l\right)}{\pi\left(\frac{\tau q^k}{T} - l\right)}d\tau \tag{13}$$

In Fig. 1 the degradation caused by WB-LTV channels due to simultaneous time and Doppler scale spread is depicted.

IV. System Development & Channel Equalization

The proposed communication scheme is based upon the simplified discretized channel model given in (12) as developed in section III. To exploit the inherent multipath time-scale diversity present in the WB-LTV channel, subcarriers which are orthogonal both in scale and time are needed [19]. If $\psi_{m,n}(t)$ represents the subcarrier at scale index m and time index n, then it should obey:

$$\langle \psi_{m,n}(t), \psi_{m',n'}(t) \rangle = \delta[m-m']\delta[n-n'] \qquad (14)$$

A family of functions satisfying (14) which instantly come into mind is the orthogonal wavelets [21]. In particular, the Meyer-Walter type of wavelets [21: p51] has good localization in all three domains, i.e. time, frequency and Mellin, and thus is a good candidate for acting as subcarriers. One of the Meyer-Walter scaling function and mother wavelet is:

$$\phi(t) = \frac{\sin\frac{(1-\rho)\pi t}{T_0} + \sin\frac{(1+\rho)\pi t}{T_0}}{\left(\frac{2\pi t}{T_0}\right)\left(1 - \frac{2\rho t}{T_0}\right)}$$

$$\psi(t) = \frac{\sin\frac{2(1-\rho)\pi t}{T_0} + \sin\frac{2(1+\rho)\pi t}{T_0}}{\left(\frac{2\pi t}{T_0}\right)\left(1 + \frac{4\rho t}{T_0}\right)}$$

$$\qquad (15)$$

$$- \frac{\sin\frac{(1-\rho)\pi t}{T_0} + \sin\frac{(1+\rho)\pi t}{T_0}}{\left(\frac{2\pi t}{T_0}\right)\left(1 - \frac{2\rho t}{T_0}\right)}$$

with $0 \le \rho \le 1/3$ and it is essentially limited support in frequency domain to $1/T_0$ and in Mellin domain to $1/\ln 2$, i.e. $q = 2$. Its associated filter coefficients are given by:

$$h[n] = \frac{1}{\sqrt{2}}\phi\left(\frac{n}{2}\right)$$
$$g[n] = (-1)^{n-1}h[1-n] \qquad (16)$$

These have interesting property like fast decay rate of $O(t^2)$ and satisfying Nyquist ISI criterion, as its autocorrelation is the Raised Cosine function [21].

A. Transmitter

Consider a block of transmitted signal which consists of $D = (2^M - 1)N$ data symbols $x_{m,n} \in \mathcal{C}$ taken from some real valued constellation $\mathcal{C}$ with $n = 0, 1, \dots, 2^m N - 1$ and $m = 0, 1, \dots, M-1$. The symbol $x_{m,n}$ is modulated by wavelet at scale index m and time index n, i.e. $\psi_{m,n}(t) = 2^{r+1}\psi(2^m t - nT_0)$. In order to avoid inter-block interference (IBI) zero padding of length $2^m Z$ for each scale m is added, with $Z \ge L_d(0)$, i.e. $x_{m,n} = 0, \forall \, 2^m N \le n < 2^m(N+Z)$.
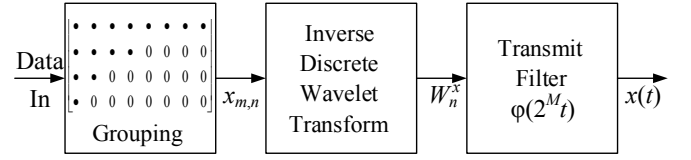
Figure 2.   Structure of the proposed transmitter. Here ● denotes the data symbols and 0 reprsents the appropriate zero padding.

Finally, one block of transmitted signal with zero padding can be represented as:

$$x(t) = \sum_{m=0}^{M-1} \sum_{n=0}^{2^m(N+Z)-1} x_{m,n}\psi_{m,n}(t) \qquad (17)$$

A visualization of the transmission scheme is portrayed in Fig. 1 depicting IBI avoidance and providing insight into the achieved spectral efficiency of approximately $2(1 - 2^{-M})\log_2|\mathcal{C}|\, N/(N+Z)$ bits/s/Hz.

Based on the following property of wavelets (MRA) [21]:

$$\phi(t) = 2^{r+1}\sum_n h[n]\phi(2t - nT_0)$$
$$\psi(t) = 2^{r+1}\sum_n g[n]\phi(2t - nT_0)$$

$$\qquad (18)$$

a computationally and memory efficient manner of implementing (17) is to perform M level inverse fast wavelet transform (FWT) [22] on $x_{m,n}$ using filter coefficients given in (16). Then a pulse shaping filter $\phi(2^M t)$ is applied on the resulting 1-D discrete time sequence. This way only one waveform needs to be stored and all pulse shaping operation are carried out in an efficient manner using FWT algorithm.

B. Receiver

Thanks to linearity of the channel the received signal can be expressed as:

$$y(t) = \sum_{k=0}^{L_s}\sum_{m=0}^{M-1}\sum_{l=0}^{L_d(m+k)}\sum_{n=0}^{2^m N-1} \tilde{h}_s[k]\tilde{h}_d[k,l]x_{m,n}\psi_{m+k,n+l}(t)$$
$$+ v(t) \qquad (19)$$

The optimal receiving strategy for a signaling scheme of (17) and (19) is to employ a bank of matched filter and sampling at the correct instant. The output of matched filter at scale index m and time index n is given by:

$$y[m,n] = \int_0^{(N+Z)T_0} y(t)\psi_{m,n}^*(t)dt + v_{m,n} \qquad (20)$$

for $n = 0, 1, \dots, 2^m N + L_d(m) - 1$ and $m = 0, 1, \dots, M + L_s - 1$. Substituting (19) into (20) yields:
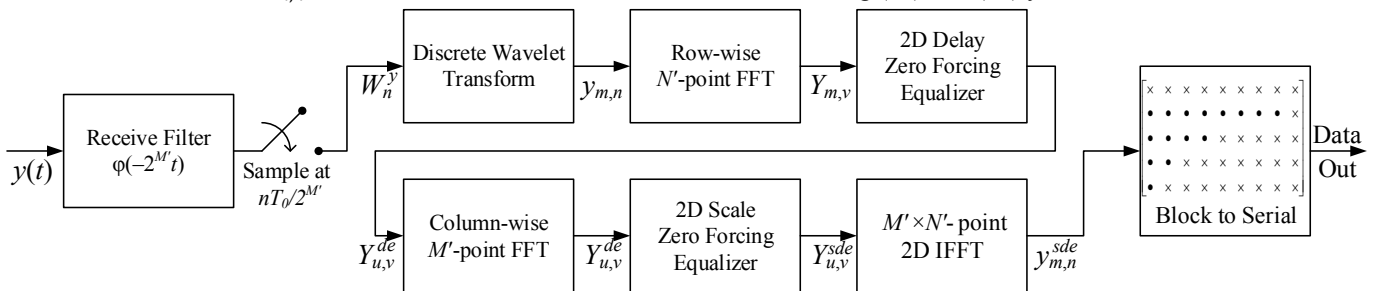
Figure 3.   Structure of the proposed transmitter. Here ● denotes the data symbols and × reprsents don't care results.

$$y[m,n] = \sum_{k=0}^{L_s} \sum_{l=0}^{L_d(m)} \tilde{h}_s[k]\tilde{h}_d[m,l]x_{m-k,n-l} + v_{m,n} \qquad (21)$$

An alternative and computationally and memory efficient method is to replace the matched filter bank with a single filter $\phi(-2^{M+L_s}t)$ (note the use of $\phi(-2^{M+L_s}t)$ instead of $\phi(-2^M t)$) and sample the output at every $T_0/2^{M+L_s}$ instant. Executing a $M + L_s$ level FWT using filter coefficients given in (16) on the resultant 1-D discrete time sequence yields $y[m,n]$, based on the MRA property (18).

C. Channel Equalization

The matched filter outputs $y[m,n]$ can be regarded as a $(M + L_s) \times 2^{M-1}(N + 2^{L_s}Z)$ block by appropriately zero padding. For notational simplicity define two new terms: $M' = M + L_s$ and $N' = 2^{M-1}(N + 2^{L_s}Z)$. This enables to treat $y[m,n]$ as an image and opens the avenue to the image processing tools. Taking row-wise discrete Fourier transform of the zero padded $y[m,n]$:

$$Y[m,v] = \left(\sum_{l=0}^{L_d(m)} \tilde{h}_d[m,l]e^{-\frac{j2\pi lv}{N'}}\right)\left(\sum_{k=0}^{L_s} \tilde{h}_s[k]X_{m-k,v}\right) + v'_{m,v}$$
$$= \tilde{H}_d[m,v]\left(\sum_{k=0}^{L_s} \tilde{h}_s[k]X_{m-k,v}\right) + v'_{m,v} \qquad (22)$$

By processing further with just a one-tap zero forcing equalizer (i.e. division by only a complex number $\tilde{H}_d[m,v]$) will give back the signal which has been compensated for delay spread. Then taking column-wise discrete Mellin transform of the equalized signal $Y_{de}[m,v]$:

$$Y_{de}[u,v] = \left(\sum_{k=0}^{L_s} \tilde{h}_s[k]q^{k(r+1)}e^{\frac{j2\pi ku}{M'}}\right)X_{u,v} + v''_{u,v}$$
$$= \tilde{H}_s[u]X_{u,v} + v''_{u,v} \qquad (23)$$

The form of (23) allows removal of residual Doppler scale spread by filtering only with a one-tap zero forcing equalizer (i.e. division by only a complex number $\tilde{H}_s[u]$). Thus taking column-wise inverse discrete Mellin transforms and row-wise inverse Fourier transform of equalized signal $Y_{sde}[u,v]$ yields back the data symbols:

$$y_{sde}[m,n] = x_{m,n} + v'''_{m,n} \qquad (24)$$

with an additive noise term. Finally the block diagram of the complete transmitter and receiver structure is shown in Fig. 2 and Fig. 3 respectively.

D. Discusions

The proposed communication scheme leads to systemic advantages as compared to related works in this field. For example, a popular method to equalize for Doppler scale is to buffer an entire frame (not just a block) of the received signal and then resample to correct for the scaling [1, 23]. The proposed method relieves the receiver hardware from large memory requirements and reduces processing delays. Another class of system is based on the time-scale RAKE receivers [6, 7]. However time-scale RAKE receivers require at least $(M + L_s) \times O(2^{M-1}N + L_d(M - 1))$ correlators, which is computationally quite intensive. The proposed scheme considerably reduces the computational complexity by judiciously replacing them by highly efficient FFT algorithm and $O(M' \times N')$ complex divisions, while maintaining the same spectral efficiency. Also, as compared to a similar OWDM based scheme over WB-LTV channel of [6], in our system the data blocks are well localized in time and are not interleaved or spread over large time intervals. Our scheme employs Meyer-Walter wavelets which decay much faster, at rate of $O(t^2)$, than the Shannon wavelet used in [6] which decays at $O(t)$, and hence easier to implement [21]. Further utilization of zero-padding for IBI prevention saves transmitting power as compared to use of cyclic prefix [24].

V. SIMULATION RESULT

The performance of proposed communication system are evaluated on an underwater acoustic mobile channel [22] which can be classified as a WB-LTV channel, where the speed of the acoustic carrier is approximately 1500 ms^{-1} and communication occurs between rapidly moving subjects, such as autonomous underwater vehicle and in presence of rapidly moving scatters with speeds up to 30 ms^{-1}. This results in typical delay to be limited to 0.8 ms and Doppler scale to range from about 0.96 to 1.04. To fit into assumption made in section III, i.e. to maintain $\eta \in [1, \eta_{max}]$, the received signal is always scaled by any constant factor larger than $1/0.96$, say 2. In digital domain this implies sampling at double the rate, which makes the Doppler scaling range a subset of the desired range. In this work, it is assumed that the receiver has perfect channel state information (CSI).

The simulation parameters chosen are: $r = 0$, $\rho = 0.25$, $D = 840$, $M = 3$, $N = 120$ and $T_0 = 0.4$ ms similar to [6, 19]. This results in $L_s = \lceil \ln 2.08 / \ln 2 \rceil = 2$ and $L_d(0) = \lceil 0.8/0.4 \rceil = 2 = Z$ and a data rate of 17.21 kbps with BPSK, 34.43 kbps with 4-PAM and 51.64 kbps with 8-PAM. The channel coefficient $\tilde{h}_s[k]$ and $\tilde{h}_d[m,l]$ are modelled as iid corresponding to brick spreading function [18]. Transfer function of a realization of such a channel is shown in Fig. 4.
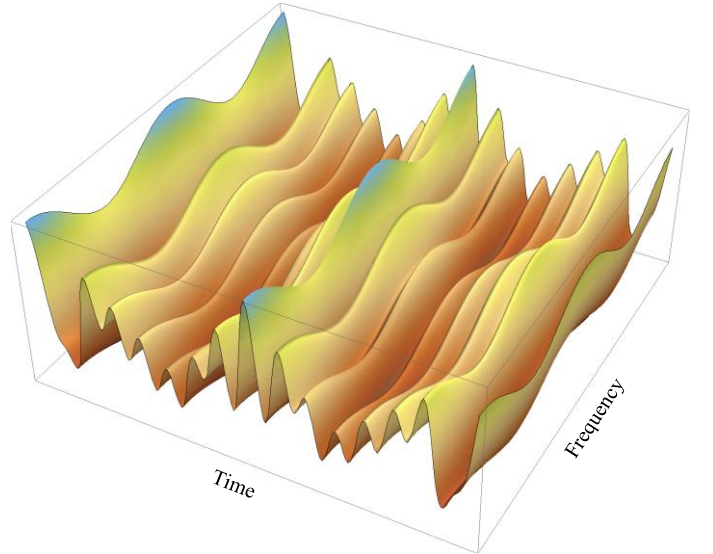

Figure 4. Transfer function of a realization of the channel at highest scale.
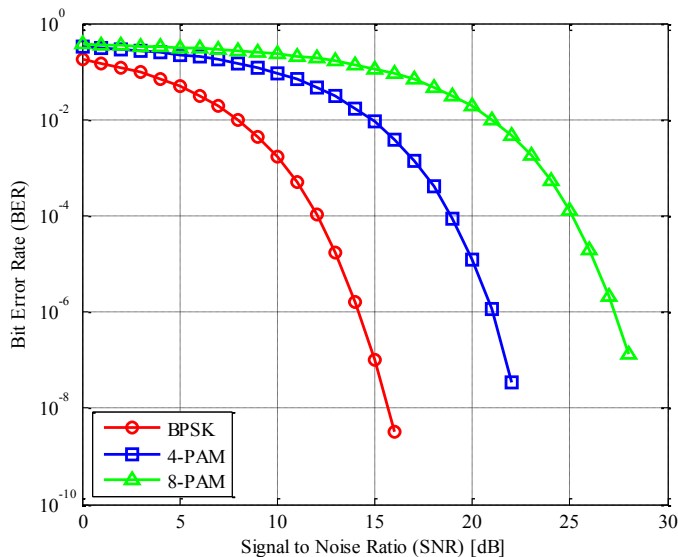
Figure 5. Simulated performace of the proposed system

Performance evaluation is carried out by simulations and the resulting BER is plotted in Fig. 5. Comparing with results reported in [5, 6, 19] clear performance gain is observed.

VI. CONCLUSION

In this paper, an inspiration from mammalian auditory system regarding scale and lag invariance is successfully transformed into an equalization technique for communication over WB-LTV channels. An OWDM based spectrally efficient signaling scheme and corresponding computationally attractive receiver structure has been proposed which takes advantage of inherent multipath time-scale diversity present in the WB-LTV channel. The systemic and performance enhancement over currently reported systems have been established using analytical as well as simulation results.

Immediate future works involves implementing channel estimation and synchronization scheme for the proposed system. In the present work, perfect transmitter receiver synchronization and CSI has been assumed (this assumption is popular for initial work, like [1, 5, 6, 17]). After which we believe that the proposed communication system has the potential to be a viable alternative to the current communication systems over WB-LTV channels.

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